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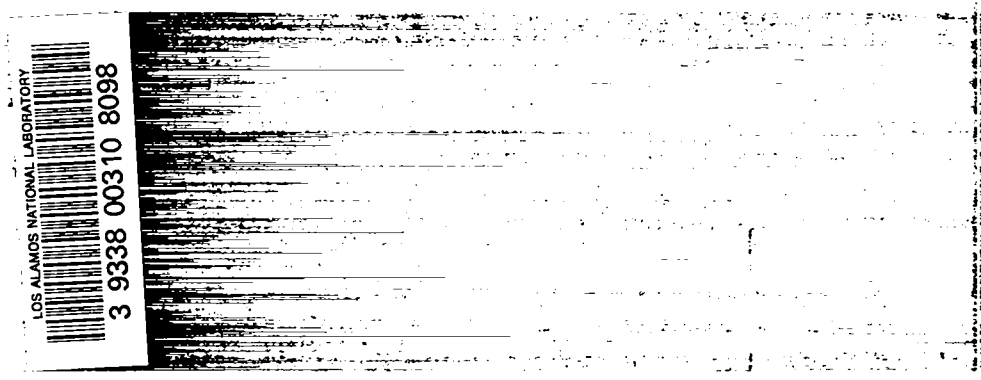

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Report written:
June 1953

LA-1560

Report distributed: AUG 20 1953

THE DETERMINATION OF NUCLEAR TEMPERATURES BY MEANS OF THE EVAPORATION MODEL

by

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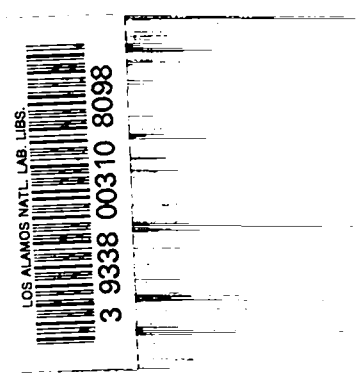

PHYSICS

ABSTRACT

Recently published data on the energy distribution of (n,n) and (n,2n) neutrons, resulting from the irradiation of various elements by 14-Mev neutrons, have been used to evaluate the constant in the exponential of the Maxwellian which describes the distribution in energy of first emitted neutrons. For each element, the total cross section for the emission of neutrons is compared with the experimental value for the sum of the inelastic and (n,2n) cross sections.

The statistical model of the nucleus, as developed by Weisskopf and co-workers,[1,2] would appear to suggest a number of more or less convenient methods for determining nuclear temperatures, T, for elements of mass number, $A \geq 50$. However, values of T deduced from (a) the analysis of neutron emission spectra[3,4] are not in agreement with values deduced (at comparable excitation energies) from (b) the shape of (n,2n) excitation curves,[5-8] and (c) cross section ratios of (α,n) and $(\alpha,2n)$ reactions,[9] T-values deduced from (b) and (c) being considerably higher than those deduced from (a).

In view of the above indicated discrepancy, we have undertaken to re-evaluate a recent series of neutron spectrum measurements[4] by utilizing recent experimental data on (n,2n) cross sections[10-13] and neutron binding energies,[14] to take account of second neutrons from (n,2n) reactions. As pointed out in Reference 4, the presence of such neutrons has the effect of lowering the apparent nuclear temperatures.

According to the statistical theory of nuclear reactions[2] the energy distribution of emitted neutrons is given by:

$$F(E_n)dE_n = \text{Const.} \quad \sigma_c E_n \, \omega(E)dE_n \quad , \tag{1}$$

where $F(E_n)$ is the number of emitted neutrons of energy between E_n and $E_n + dE_n$,

σ_c is the cross section for the formation of a compound nucleus $A + 1$ from the level of the residual nucleus A reached in the reaction. For neutrons, σ_c is assumed to be essentially constant and approximately equal to the inelastic collision cross section, σ_i .

$\omega(E)$ is the level density in the residual nucleus at an excitation energy $E = E_o - E_n$, E_o being the incident neutron energy.

By developing $\ln \omega(E)$ about E_o, and making use of the thermodynamic analogies wherein $\ln \omega$ is identified with the entropy, S(E), and $dS/dE = 1/T(E)$ (T already contains the Boltzman constant and has the dimensions of an energy), one obtains the relation:

$$\frac{F(E_n)}{E_n} \cong \text{Const.} \quad \sigma_c \, e^{-E_n/T(E_o)} \tag{2}$$

and $T(E_o)$ may be interpreted as the temperature of the residual nucleus at excitation energy E_o. The above expansion assumes that E_n is small compared to E_o and this is indeed borne out by experiment, i.e., the residual nucleus is left at an excitation energy of approximately E_o after most "first neutron" emissions. Moreover, Equation 2 fits the experimental data on neutron emission spectra[3,4] remarkably well.

To obtain an expression for the level density function $\omega(E)$, one may proceed as follows:[2] It is first of all postulated that E can be expanded in powers of T about T = 0, that the third law of thermodynamics ($dE/dT = 0$ at $T = 0$) is applicable, and that terms higher than second order are negligible. Under these conditions,

$$E = aT^2 \tag{3}$$

thus relating temperature to excitation energy for any given nucleus. We now can evaluate the level density function, ω, in terms of a and E, i.e., $dS = dE/T = 2adT$; $S = \ln \omega = 2aT +$ Const., giving

$$\omega(E) = \text{Const. } e^{2aT} = \text{Const. } e^{(4aE)^{1/2}} \tag{4}$$

The method adopted for subtracting out the second neutrons from the data of Reference 4 is as follows: The data of References 10-13 were used to make a plot of $\sigma(n,2n)$ vs A, parametric in Q. The value of $\sigma(n,2n)$ for any isotope or combination of isotopes of given A and Q, both A and Q being weighted according to isotopic abundance, is read off from the curve, making use of interpolation where necessary. From this value and the inelastic collision cross section,[15] σ_i, one obtains the ratio $\sigma_i/\sigma(n,2n) = K$.

Equation 2 now permits us to equate total numbers of first and second neutrons, the first neutrons being emitted from compound nucleus A + 1, the second neutrons being emitted from nucleus A, i.e.,

$$\int_0^{E_o} F(E_{n_1})dE_{n_1} = K \int_0^{E_o - E_b - 2T_1} F(E_{n_2})dE_{n_2} , \tag{5}$$

E_b represents the binding energy of the last neutron in nucleus A (see Fig. 1) and $2T_1$ is the average energy of the neutrons emitted by nucleus A + 1. $F(E_{n_1})$ and $F(E_{n_2})$ are, respectively, the numbers of first and second neutrons emitted in the energy interval between E_n and $E_n + dE_n$. If we now define

$$R = \frac{F(E_{n_2})}{F(E_{n_1})} = \frac{C_2 E_{n_2} \exp.\left[-E_{n_2}/T_2\right]}{C_1 E_{n_1} \exp.\left[-E_{n_1}/T_1\right]} \tag{6}$$

for $E_n \leq E_o - E_b - 2T_1$, we see that $F(E_{n_1}) = \left[1/(1 + R)\right] F(E_n)$, where $F(E_n)$ is the

number of observed first and second neutrons in the energy interval dE_n.

We must now proceed to the evaluation of R. By integrating both sides of Equation 5 we obtain $C_1/C_2 = K\,(T_2^2 / T_1^2)$. T_2 is evaluated by using Equation 3, i.e., $E_o = aT_1^2$ and

$$E_o - E_b - 2T_1 = aT_2^2 \quad . \tag{7}$$

We then have

$$R = \frac{1}{K}\left(\frac{T_1}{T_2}\right)^2 \exp.\left[-E_n(T_1 - T_2)/T_1\,T_2\right] \tag{8}$$

As a first approximation T_1 is taken equal to T (Reference 4). The calculation is then repeated using the new value of T_1 to obtain better values for both T_1 and T_2.

In Table 1 are given the corrected values of T_1 and a (Equation 4). T_1 is given by $\partial/\partial E_n \left\{\ln\left[F(E_n)/\,(1 + R)E_n\right]\right\}^{-1}$ as obtained from a plot of $\ln\left[F(E_n)/(1 + R)E_n\right]$ vs E_n, while a is given by the slope, S, of a plot of $\ln\left[F(E_n)/(1 + R)E_n\right]$ vs $E^{1/2}$; i.e., $a = S^2/4$. The corrected values of T_1 are still almost a factor of 2 lower than the values obtained by methods (b) and (c).

If the above corrections for (n,2n) neutrons are valid, then one should be able to obtain the total cross section for emission of neutrons due to inelastic interactions by adding to the number of observed neutrons for each element

$$\int_0^{0.5\ \text{Mev}} \left[F(E_{n_1}) + F(E_{n_2})\right] dE_n \quad . \tag{9}$$

The values thus obtained, together with $2\sigma(n,2n) + \sigma(n,n)$, are given in Table 1, rows 3 and 4, respectively. $\sigma(n,n)$ is taken equal to $\sigma_i - \sigma(n,2n)$; the errors shown do not include errors introduced by Equation 7. In only one case is the disagreement outside of the estimated experimental uncertainty. In the evaluation of $\sigma(n,n)$, cross sections for (n, charged particle) reactions are neglected, although they are not always completely negligible.[10] The values in row 3 should therefore always be less than the values in row 4.

On the basis of the model under discussion, it is very difficult to see how the direct measurement of neutron spectra can be interpreted to give significantly higher values of T than are presented in Table 1. There are a number of disturbing considerations. First of all there is, of course, the possibility that some of the emitted neutrons do not arise from an evaporation process, but rather through a direct interaction with the incident neutron. This

implies that the Bohr assumption (compound nucleus formation, rapid sharing of energy among the nucleons and finally de-excitation by particle or X-ray emissions), which forms the basis of the statistical model, is simply not valid for all interactions. However, this would tend to increase our apparent nuclear temperatures. Second, there is the quite likely possibility that σ_c (Equation 1) is not energy independent. Third, the approximately 4 Mev of excitation energy of the residual nucleus, after an (n,2n) reaction, may not be sufficiently high to permit the statistical treatment we have used here to take account of second neutrons from (n,2n) processes. Precisely how the above two uncertainties might effect the deduced apparent temperature values is difficult to predict. Finally there exists the complication of energy degradation of the neutrons from both (n,n) and (n,2n) processes by a second inelastic scattering. However, that this effect could significantly alter our T values is unlikely for the reasons given in Reference 4.

As for the T values deduced from (α,n), $(\alpha,2n)$ and (n,2n) reactions, it might be said that they do not suffer from the difficulty of correction for second neutrons. There is the additional assumption involved that two neutrons are always emitted when it is energetically possible; i.e.,

$$\frac{\sigma(n,n)}{\sigma_i \cong \sigma(n,2n) + \sigma(n,n)} \cong \frac{\int_{E'}^{E_o} E_n\, e^{-E_n/T}\, dE_n}{\int_{0}^{E_o} E_n\, e^{-E_n/T}\, dE_n} \qquad (10)$$

where E' is given in Fig. 1. We then have

$$\sigma(n,2n) = \sigma_i \left[1 - \left(1 + \frac{E'}{T}\right) e^{-E'/T}\right] \quad , \qquad (11)$$

which is the equation used for determination of T values from (n,2n) data. Although a small change in E' would markedly effect T when $E' \cong T$, it is unlikely that E' can be changed significantly since competition from γ emission, when emission of a second neutron is energetically possible, is probably not serious. It is, of course, possible that the thresholds for (n,2n) reactions contain somewhat of a systematic error since $\sigma(n,2n)$ goes as $(E')^2$ near threshold. (By expanding the exponential in Equation 11, it is seen that, to a first approximation, $\sigma(n,2n) = E'^2/T^2$.) However, on the basis of this argument, one would expect the thresholds to go down and E' to increase, and these are the wrong directions to account for the apparent discrepancies.

In a recent attempt to account for the discrepancies between values for nuclear temperatures obtained by different methods, B. L. Cohen[5] suggests that the measured energy distribution of emitted particles might be expected to go as

$$\frac{1}{\Sigma} = -\frac{\partial \ln \eta}{\partial E} + \frac{1}{T(E_o - E)} \qquad (12)$$

where

$$\Sigma = \left[-\frac{\partial \ln \frac{F(E_n)}{\sigma_o E_n}}{E_n} \right]^{-1}$$

and η is the "sticking probability" (which in our analysis we have assumed to be unity) and is defined as σ_c/σ_o, σ_o being the cross section for the formation of a compound nucleus A + 1 from the ground state of nucleus A, and σ_c being previously defined.

Cohen's conclusion that both η and $\partial\eta/\partial E$ decrease as E decreases implies (from Equation 12) that the true temperature values, as obtained from neutron spectrum measurements for E_o = 14 Mev, are even lower than those given in Table 1.

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TABLE 1

Summary of Results

Element	Fe	Cu	Zn	Ag	Cd	Sn	Pb	Bi
T_1 (Mev)	.78±.08	.90±.09	.78±.08	.66±.06	.77±.08	.56±.06	1.0±.11	1.3±1.4
$a\,(\text{Mev})^{-1}$	20±4	19±4	28±6	29±6	22±4	42±8	15±3	11±2
$\int_0^{12\ \text{Mev}} \sigma(E_o, E_n)\,dE_n$ (barns)	1.6±.4	1.3±.3	1.7±.4	2.1±.5	2.7±.7	2.6±.7	3.9±.9	4.3±1.1
$\sigma_i + \sigma(n, 2n)$ (barns)	1.6	2.1	1.7	2.3	2.6	2.8	4.4	4.4

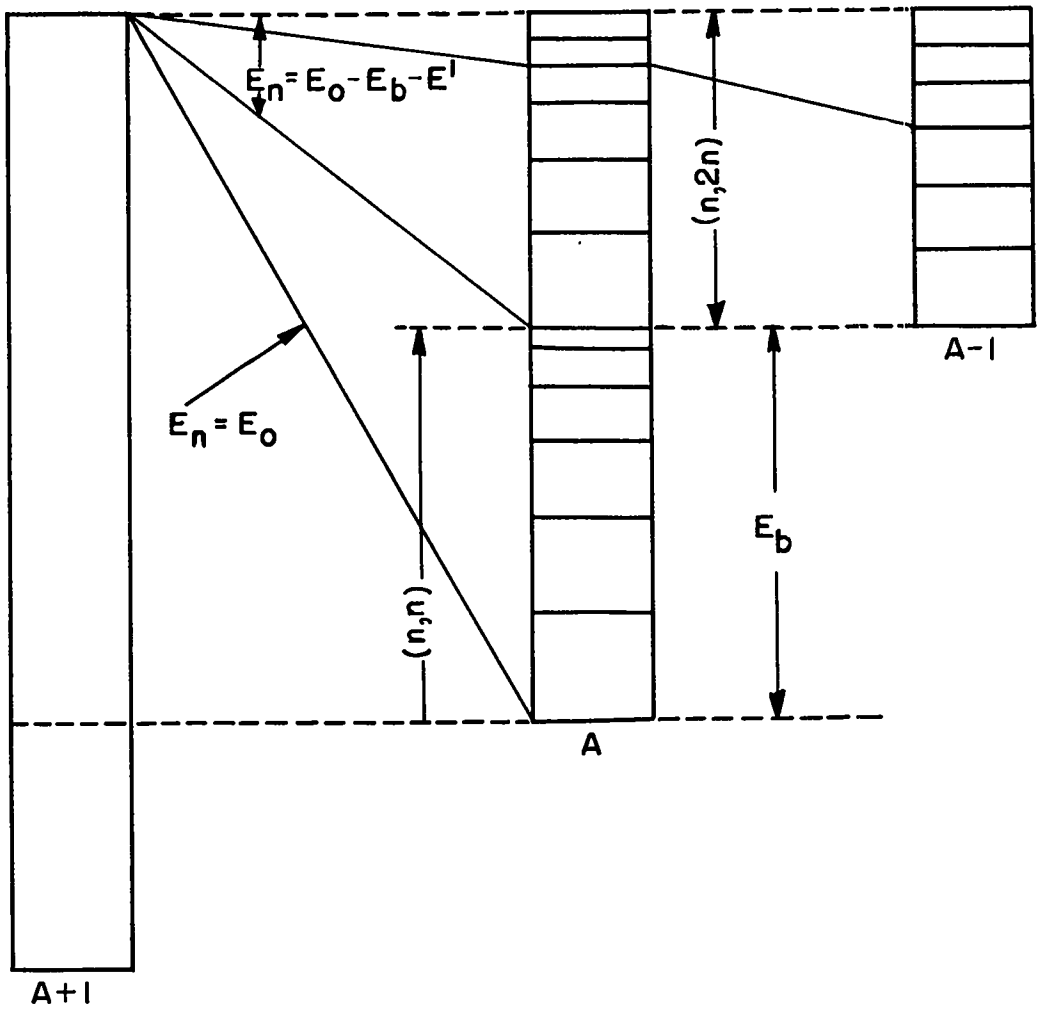

Fig. 1. Permissible energy values for neutrons from (n, n) and (n, 2n) reactions